

Discussion: “Narrative Restrictions and Proxies”
by Giacomini, Kitagawa & Read

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Outline

- ① Robustness of narrative proxy approach
- ② Inference with a weak proxy
- ③ Conclusion

Bivariate SVAR model with narrative signals

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}, \quad t = 1, \dots, T.$$

- Shocks $(\varepsilon_{1t}, \varepsilon_{2t})$ are i.i.d. mean zero, variance 1, mutually independent.
- **Narrative signals** about ε_{1t} (for simplicity, no info about ε_{2t}):

$$Z_t = \begin{cases} 1 & \text{if we believe } \varepsilon_{1t} > 0, \\ 0 & \text{if sign unknown,} \\ -1 & \text{if we believe } \varepsilon_{1t} < 0. \end{cases}$$

- Data: (y_{1t}, y_{2t}, Z_t) . Shocks are unobserved.
- Identified set for impulse responses Θ_{ij} is large if we ignore Z_t . How to model+exploit Z_t ?

Bayesian inference when signals are perfect

- Assume first that **signals are perfect**:

$$\text{sign}(\varepsilon_{1t}) = Z_t \quad \text{whenever } Z_t \neq 0.$$

- This implies restrictions that can substantially sharpen inference about impulse responses: [Antolín-Díaz & Rubio-Ramírez \(2018\)](#); [Ludvigson, Ma & Ng \(2019\)](#); [Giacomini, Kitagawa & Read \(2021\)](#)

$$\text{sign} \left(\frac{\Theta_{22}y_{1t} - \Theta_{12}y_{2t}}{\Theta_{11}\Theta_{22} - \Theta_{12}\Theta_{21}} \right) = Z_t \quad \text{whenever } Z_t \neq 0.$$

- Subjective/robust Bayesian inference based on these restrictions imposes strong assumptions:
 - Signals are **perfect** and **arrive randomly** (likelihood function appears to impose that the event $\{Z_t \neq 0\}$ is independent of the shocks).
 - SVAR model assumes shocks are **invertible** (functions of only current and past data).

Proxy approach

- More generally, we could assume that the signal Z_t is a potentially **imperfect proxy**:

$$Z_t = F(\varepsilon_{1t}, u_t), \quad u_t \perp\!\!\!\perp (\varepsilon_{1t}, \varepsilon_{2t}),$$

where $F: \mathbb{R}^2 \rightarrow \{-1, 0, 1\}$ is unknown, and u_t is unobserved measurement error.

- Example: $Z_t = \text{sign}(\varepsilon_{1t} + u_t) \times \mathbb{1}(|\varepsilon_{1t}| \geq 2)$.
- Yet, we assume that signals are not too inaccurate overall:

$$\text{Cov}(Z_t, \varepsilon_{1t}) > 0.$$

Proxy approach: Robust estimation

$$Z_t = F(\varepsilon_{1t}, u_t), \quad \text{Cov}(Z_t, \varepsilon_{1t}) > 0$$

- Consider estimation of the relative IR $\theta \equiv \Theta_{21}/\Theta_{11}$. Since

$$y_{2t} = \theta y_{1t} + (\Theta_{22} - \Theta_{21} \frac{\Theta_{12}}{\Theta_{11}}) \varepsilon_{2t},$$

we can estimate θ by 2SLS regression of y_{2t} on y_{1t} , with Z_t as IV.

Romer & Romer (1989); Hamilton (2003); Budnik & Rünstler (2020)

- Appealing robustness properties:
 - ① IV exclusion restriction holds even under **misclassification** and **non-random signal arrival**.
 - ② Can allow shocks to be non-invertible with 2SLS version of Local Projection (or recursive VAR with Z_t ordered first). Stock & Watson (2018); Plagborg-Møller & Wolf (2021)

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Proxy approach: Weak identification

- Credible signals are usually only available for a small number K of periods \implies proxy Z_t is mostly zero \implies **weak identification**.
- G, K & R helpfully point out that the standard weak-IV robust SVAR procedure does *not* work if we model K as finite asymptotically. **Montiel Olea, Stock & Watson (2021)**
 - This procedure requires asy. normality of reduced-form sample moments.
 - But $T^{-1/2} \sum_{t=1}^T Z_t y_{1t} \xrightarrow{p} 0$, since $Z_t = 0$ for all but finite no. of obs.
 - Can fix the procedure if shocks are Gaussian, but this seems fragile.
- Should we therefore give up on the robustness afforded by the proxy approach?

Proxy inference in small samples

- Fundamentally, the setting with a sparse proxy Z_t is a **small-sample** problem. Can we apply weak-IV procedures that are geared towards small samples?

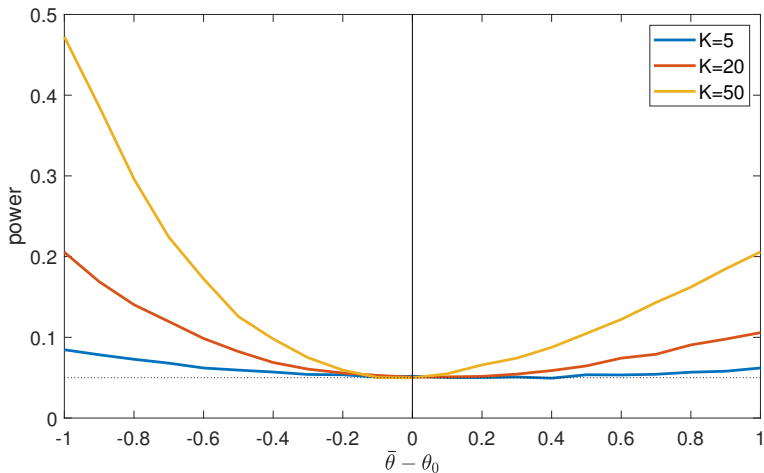
- **Permutation Anderson-Rubin test:** Under $H_0: \theta = \bar{\theta}$,

$$y_{2t} - \bar{\theta}y_{1t} = (\Theta_{22} - \Theta_{21}\frac{\Theta_{12}}{\Theta_{11}})\varepsilon_{2t} \implies (y_{2t} - \bar{\theta}y_{1t}) \perp\!\!\!\perp Z_t.$$

Fisher permutation test of independence: **Imbens & Rosenbaum (2005)**

- ① Compute $|\widehat{\text{Corr}}(y_{2t} - \bar{\theta}y_{1t}, Z_t)|$.
 - ② Compute same statistic over all possible permutations of the IV data points Z_1, \dots, Z_T .
 - ③ Reject if original statistic exceeds 95th percentile of permutation distribution.
- **Exact size** in finite samples. Does not require shocks to be Gaussian.

Simulated size/power of nominal 5% permutation test



DGP: Same as G, K & R, except perfect signals arrive in first K periods where $\varepsilon_{1t} > 0$ (so uncond'l shock distribution is normal). $T = 500$. 5,000 simulations. 1,000 random permutations.

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Conclusion

- G, K & R are doing important work on understanding+improving narrative identification.
- Very helpful to point out the small-sample problem that (i) causes a weak proxy issue and (ii) cannot be tackled with the usual inference procedure based on asy. normality.
- My opinion: It would be a shame to give up on the proxy approach, as it is robust to misclassification, non-random signal arrival, and non-invertibility.
- My suggestion: Use small-sample weak-IV robust procedures, such as permutation test.
 - Power may be low, which seems inevitable given nature of restrictions.
 - May want to relax shock independence assumptions (also true for other approaches).