Advanced Econometrics: Time Series Models

Syllabus: Version 2h (October 23, 2017)

Instructors:	$Mikkel \ Plagborg-Moller, \ mikkelpm@princeton.edu$
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Lectures:	Mon/Wed 10.40am–12.10pm, JRR 198
Office hours:	Wed 2.30–3.45, JRR 282 (Plagborg-Moller)

Description. Concepts and methods of time series analysis and their applications to economics. Time series models to be studied include simultaneous stochastic equations, VAR, ARIMA, and state-space models. Methods to analyze trends, second-moment properties via the autocovariance function and the spectral density function, methods of estimation and hypothesis testing and of model selection will be presented. Kalman filter and applications as well as unit roots, cointegration, ARCH, and structural breaks models are also studied.

Prerequisites. ECO 517 and 518, or equivalent. Students from outside the Economics PhD program should contact the instructors to obtain permission to take the course.

Material. The course material is self-contained and there is no required textbook for the course. Handouts covering most of the material will be made available on the website. Some students might find it useful to have a textbook as an additional reference. Good reference books are:

- Brockwell, P. J., and Davis, R. A. (1991). *Time Series: Theory and Methods*. 2nd edition, Springer. (A beautiful mathematical treatment of the classic theory of covariance stationary time series, but not aimed at economists.)
- Davidson, J. (1994). *Stochastic Limit Theory*. Oxford University Press. (Thorough, technical treatment of stochastic limit theory for dependent data.)
- Hall, P., and Heyde, C. C. (1980). *Martingale Limit Theory and Its Application*. Academic Press. (Well-written technical treatment of martingale asymptotics.)

- Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press. (Comprehensive reference for time series econometrics methods developed before the mid-1990s.)
- Hayashi, F. (2000). *Econometrics*. Princeton University Press. (Accessible treatment of GMM and stochastic limit theory for time series data.)
- Herbst, E. P., and Schorfheide, F., (2015). *Bayesian Estimation of DSGE Models*. Princeton University Press. (Recent reference on computational methods for Bayesian inference in nonlinear structural time series models.)
- Kilian, L., and Lütkepohl, H. (2017). *Structural Vector Autoregressive Analysis*. Cambridge University Press, forthcoming. (Recent reference on VAR and SVAR methods for applied macro.)
- Lütkepohl, H. (2005). New Introduction to Multiple Time Series Analysis. Springer. (Comprehensive treatment of estimation and inference in VARIMA models.)

This syllabus also includes a list of additional readings that are useful for a deeper understanding of the material. Many of these readings are available electronically.

Homework. Problem sets will be posted on the course website every one or two weeks. The due date will typically be one week after the assignment is posted. Problem sets should be printed out and handed in at the beginning of class on the due dates. Students are encouraged to collaborate on the problem sets. However, answers should be typed up independently, and large chunks of code must not be shared among students. Problem sets will be graded coarsely, i.e., a full score will be given as long as the work demonstrates dedication and thoughtfulness. The instructors reserve the right to subtract points for sloppy exposition, including unreadable code or poor document structure. If you find a grading error, please resubmit your problem set along with a one-paragraph explanation; the instructors reserve the right to re-grade the entire problem set.

Exams. The course will feature a take-home midterm, as well as the option to take a final exam *or* write a research paper. The take-home midterm will be posted a week before fall recess and due right after fall recess. No collaboration is allowed on the midterm. The final exam will focus primarily but not exclusively on the second half of the course. Students may elect to substitute the final exam with a final course paper. The final paper can be empirical, theoretical, or a mix, but it must represent a contribution to the academic literature. Any

student who wishes to write a final paper must submit a proposal to the instructors before Thanksgiving, and the proposal must be approved by the instructors by the last day of class. The final paper is due on the same date as the final exam.

Grading. The final course grade will be a monotonic function of the weighted average of (i) the average problem set score (30% weight), (ii) the midterm score (20% weight), and (iii) the final exam score or final paper score (50% weight).

Code of conduct. All course activities, including class meetings and homework assignments, are subject to the university's academic code and code of conduct as detailed in the "Rights, Rules, Responsibilities" publication.

Accommodations for students with disabilities. Students needing accommodations because of a documented disability should notify the instructors as soon as possible. Failure to do so may result in the instructors' inability to respond in a timely manner. All discussions will remain confidential, although the instructor may contact the Office of Disability Services to discuss appropriate implementation.

Important dates. These dates are preliminary. Changes will be announced via course email.

Sep 13 (Wed): First class with M. Plagborg-Moller

Oct 23 (Mon): Last class with M. Plagborg-Moller, take-home midterm posted

- Oct 25 (Wed): First class with C. Sims
- Oct 30 (Mon), Nov 1 (Wed): No class due to Fall recess
- Nov 6 (Mon): Midterm due in class

Nov 21 (Tue): Deadline for submission of final paper topic (optional)

Nov 22 (Wed): No class due to Thanksgiving recess

Dec 13 (Wed): Last class, deadline for approval of final paper topic (optional)

Exam period (date TBA): Final exam, due date for final paper (optional)

Outline of the first half of the course. The first half of the course (taught by M. Plagborg-Moller) will cover classic building-block tools in time series econometrics, while the second half (taught by C. Sims) will move toward the research frontier. The following outline, which covers only the first half of the course, is preliminary and may change without warning.

- 1. Stationary models.
 - i) Covariance/strict stationarity.
 - ii) Autocovariance function.
 - iii) VARMA, stationarity, invertibility.
 - iv) Impulse response functions.
 - v) Prediction, Granger/Sims causality, likelihood factorization.
 - vi) VAR estimation, inference, stationary asymptotics.
 - vii) Bayesian VARs, Bernstein-von Mises theorem.
 - viii) Wold decomposition.
 - ix) Model selection.
- 2. Spectral analysis.
 - i) Seasonality.
 - ii) Approximation of arbitrary spectrum by AR/MA.
 - iii) Periodogram smoothing.
- 3. Inference with weakly dependent data.
 - i) Central Limit Theorem, martingale difference sequences, mixing.
 - ii) Testing for serial correlation.
 - iii) Applications to GMM.
 - iv) Bootstrap.
- 4. Functional Central Limit Theorem.
 - i) Testing for structural breaks.

- 5. Long-run variance estimation.
 - i) HAC kernel estimators.
 - ii) Fixed-*b* asymptotics.
 - iii) VAR-HAC.
- 6. Non-stationary models.
 - i) I(1) processes, Beveridge-Nelson decomposition, VARIMA.
 - ii) Spurious regression.
 - iii) Bayesian vs. frequentist perspective.
 - iv) Frequentist asymptotics for unit roots, local-to-unity.
 - v) Detrending.
 - vi) Cointegration, VECM models (time permitting).
 - vii) Müller-Watson long-run inference (time permitting).

Reading list for the first half of the course

Introductory readings are listed first and marked with a star (*). Other readings are included for your reference. Original contributions are not always cited when good handbook/textbook references are available. The reading list is preliminary and may change without warning.

1 Stationary models

Models, inference, prediction

- * Hayashi: chapters 6.1–6.4.
- * Herbst and Schorfheide: chapters 3.1–3.2.
- * Lütkepohl: chapters 2–3.

Brockwell and Davis: chapters 1.1–1.5, 2.1–2.9, 3.1–3.5, 5.1–5.5, 5.7, 11.1–11.4.

- Hamilton: chapters 2–4, 10–12.
- Kilian and Lütkepohl: chapters 2, 5.
- Lippi, M., and Reichlin, L. (1994). "VAR analysis, nonfundamental representations, Blaschke matrices." *Journal of Econometrics* 63(1), 307–325.
- van der Vaart, A. W. (1998). Asymptotic Statistics. Cambridge University Press. Chapter 10.

Model selection

- Claeskens, G., and Hjort, N. L. (2008). Model Selection and Model Averaging. Cambridge University Press. Chapters 1–4.
- * Lütkepohl: chapter 4.

Brockwell and Davis: chapter 9.

Geweke, J., and Meese, R. (1981). "Estimating regression models of finite but unknown order." *International Economic Review* 22(1), 55–70.

- Hansen, B. E. (2005). "Challenges for Econometric Model Selection." *Econometric Theory* 21(1), 60–68.
- Leeb, H., and Pötscher, B. M. (2005). "Model Selection and Inference: Facts and Fiction." *Econometric Theory* 21(01), 21–59. Sections 1–2.

Applications

- Bernanke, B. S., and Kuttner, K. N. (2005). "What Explains the Stock Market's Reaction to Federal Reserve Policy?" *Journal of Finance* 60(3), 1221–1257.
- Giannone, D., Lenza, M., and Primiceri, G. E. (2015). "Prior Selection for Vector Autoregressions." *Review of Economics and Statistics* 97(2), 436–451.
- Sims, C. A. (1972). "Money, Income, and Causality." *American Economic Review* 62(4), 540–552.
- Stock, J. H., and Watson, M. W. (2003). "Forecasting Output and Inflation: The Role of Asset Prices." Journal of Economic Literature 41(3), 788–829.

2 Spectral analysis

Representation theory and inference

* Hamilton: chapter 6.

Brockwell and Davis: chapters 4, 10.1–10.5, 11.6.

Berk, N. (1974). "Consistent Autoregressive Spectral Estimates." Annals of Statistics 2(3), 489–502.

Hannan, E. J. (1970). Multiple Time Series. John Wiley & Sons. Chapters III.2–3, III.5.

Applications

- Dew-Becker, I., and Giglio, S. (2016). "Asset Pricing in the Frequency Domain: Theory and Empirics." *Review of Financial Studies* 29(8), 2029–2068.
- King, R. G., and Watson, M. W. (1996). "Money, Prices, Interest Rates and the Business Cycle." *Review of Economics and Statistics* 78(1), 35–53.

- Qu, Z., and Tkachenko, D. (2012). "Frequency Domain Analysis of Medium Scale DSGE Models with Application to Smets and Wouters (2007)." In Advances in Econometrics, Volume 28: DSGE Models in Macroeconomics – Estimation, Evaluation and New Developments, edited by Balke, N., Canova, F., Milani, F., and Wynne, M. A., Emerald Group Publishing, 319–385.
- Sala, L. (2015). "DSGE Models in the Frequency Domain." Journal of Applied Econometrics 30(2), 219–240.
- Sargent, T. J., and Surico, P. (2011). "Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals." *American Economic Review* 101(1), 109–128.
- Watson, M. W. (1993). "Measures of Fit for Calibrated Models." Journal of Political Economy 101(6), 1011–1041.

3 Inference with weakly dependent data

Abstract theory

* Hayashi: chapters 2, 6.5.

Brockwell and Davis: chapters 6–7.

Davidson: chapters 13–14, 24.

Hall and Heyde: chapter 3.

Hamilton: chapter 7.

GMM

- * Hayashi: chapters 7.1–7.4.
 - Hansen, L. P., Heaton, J., and Yaron, A. (1996). "Finite-Sample Properties of Some Alternative GMM Estimators." Journal of Business & Economic Statistics 14(3), 262– 280.
 - Hansen, L. P., and Heckman, J. J. (1996). "The empirical foundations of calibration." Journal of Economic Perspectives 10(1), 87–104.

- Kydland, F., and Prescott, E. (1996). "The computational experiment: an econometric tool." *Journal of Economic Perspectives* 10(1), 69–85.
- Newey, W. K., and McFadden, D. L. (1994). "Large sample estimation and hypothesis testing." In *Handbook of Econometrics, Volume IV*, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 36, 2111–2245.

Bootstrap

- * Kilian and Lütkepohl: chapters 12.1–12.5.
 - Brüggemann, R., Jentsch, C., and Trenkler, C. (2016). "Inference in VARs with conditional heteroskedasticity of unknown form." *Journal of Econometrics* 191(1), 69–85.
 - Gonçalves, S., and Kilian, L. (2004). "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form." *Journal of Econometrics* 123(1), 89–120.
 - Horowitz, J. L. (2001). "The Bootstrap." In *Handbook of Econometrics, Volume 5*, edited by Heckman, J. J., and Leamer, E., Elsevier, chapter 52, 3159–3228.
 - Kilian, L. (1998). "Small-sample Confidence Intervals for Impulse Response Functions." *Review of Economics and Statistics* 80(2), 218–230.

Applications

- Hansen, L. P., and Singleton, K. J. (1982). "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectation Models." *Econometrica* 50(5), 1269–1286.
- Mankiw, N. G., Reis, R., and Wolfers, J. (2004). "Disagreement about Inflation Expectations." In NBER Macroeconomics Annual 2003, Volume 18, edited by Gertler, M., and Rogoff, K., National Bureau of Economic Research, 209–248.
- Mavroeidis, S., Plagborg-Møller, M., and Stock, J. H. (2014). "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve." *Journal of Economic Literature* 52(1), 124–188.

4 Functional Central Limit Theorem

Abstract theory

Davidson: chapters 26–30.

Hall and Heyde: chapter 4.

- Andrews, D. W. K. (1994). "Empirical process methods in econometrics." In Handbook of Econometrics, Volume IV, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 37, 2247–2294.
- Phillips, P. C. B., and Solo, V. (1992). "Asymptotics for Linear Processes." Annals of Statistics 20(2), 971–1001.

Structural breaks

- Andrews, D. W. K., (1993). "Tests for Parameter Instability and Structural Change with Unknown Change Point." *Econometrica* 61(4), 821–856.
- Andrews, D. W. K. and Ploberger, W. (1994). "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative." *Econometrica* 62(6), 1383–1414.
- Bai, J. (1997). "Estimation of a Change Point in Multiple Regression Models." Review of Economics and Statistics 79(4), 551–563.
- Bai, J. (1997). "Estimating multiple breaks one at a time," *Econometric Theory* 13(3), 315–352.
- Bai, J., and Perron, P. (1998). "Estimating and Testing Linear Models with Multiple Structural Changes." *Econometrica* 66(1), 47–78.
- Bai, J., and Perron, P. (2003). "Computation and Analysis of Multiple Structural Change Models." Journal of Applied Econometrics 18, 1–22.
- Elliott, G., and Müller, U. K. (2006). "Efficient Tests for General Persistent Time Variation in Regression Coefficients." *Review of Economic Studies* 73(4), 907–940.
- Müller, U. K. and Petalas, P.-E. (2010). "Efficient Estimation of the Parameter Path in Unstable Time Series Models." *Review of Economic Studies* 77(4), 1508–1539.

- Nyblom, J. (1989). "Testing for the Constancy of Parameters Over Time." Journal of the American Statistical Association 84(405), 223–230.
- Pesaran, M. H., and Timmermann, A. (2007). "Selection of estimation window in the presence of breaks." *Journal of Econometrics* 137(1), 134–161.
- Stock, J. H. (1994). "Unit roots, structural breaks and trends." In Handbook of Econometrics, Volume 4, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 46, 2739–2841. Sections 2.2 and 5.

Applications

- * Hansen, B. E. (2001). "The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity." *Journal of Economic Perspectives* 15(4), 117–128.
 - Stock, J. H., and Watson, M. W. (1996). "Evidence on Structural Instability in Macroeconomic Time Series Relations." Journal of Business & Economic Statistics 14(1), 11–30.

5 Long-run variance estimation

- * Hayashi: chapter 6.6.
- * Müller, U. K. (2014). "HAC Corrections for Strongly Autocorrelated Time Series." Journal of Business & Economic Statistics 32(3), 311–322. Sections 1–3.
 - Andrews, D. W. K. (1991). "Heteroskedasticity and autocorrelation consistent covariance matrix estimation." *Econometrica* 59(3), 817–858.
 - Andrews, D. W. K., and Monahan, J. C. (1992). "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator." *Econometrica* 60(4), 953– 966.
 - Den Haan, W. J., and Levin, A. (1997). "A practitioner's guide to robust covariance matrix estimation." In *Handbook of Statistics, Volume 15*, edited by Maddala, G. S., and Rao, C. R., North-Holland, 299–342.
 - Den Haan, W. J., and Levin, A. (1998). "Vector Autoregressive Covariance Matrix Estimation." Manuscript, London School of Economics.

- Ibragimov, R., and Müller, U.K. (2010). "t-Statistic Based Correlation and Heterogeneity Robust Inference." Journal of Business and Economic Statistics 28(4), 453–468.
- Jansson, M. (2004). "The Error in Rejection Probability of Simple Autocorrelation Robust Tests." *Econometrica* 72(3), 937–946.
- Kiefer, N. M., and Vogelsang, T. J. (2002). "Heteroskedasticity–Autocorrelation Robust Testing Using Bandwidth Equal To Sample Size." *Econometric Theory* 18(6), 1350– 1366.
- Kiefer, N. M., and Vogelsang, T. J. (2005). "A New Asymptotic Theory for Heteroskedasticity–Autocorrelation Robust Tests." *Econometric Theory* 21(6), 1130–1164.
- Lazarus, E., Lewis, D. J., and Stock J. H. (2017). "The Size–Power Tradeoff in HAR Inference." Manuscript, Harvard University.
- Müller, U. K. (2007). "A theory of robust long-run variance estimation." Journal of Econometrics 141(2), 1331–1352.
- Newey, W. K., and West, K. D. (1987). "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica* 55(3), 703– 708.
- Newey, W. K., and West, K. D. (1994). "Automatic Lag Selection in Covariance Matrix Estimation." *Review of Economic Studies* 61(4), 631–653.
- Pötscher, B. M. (2002), "Lower Risk Bounds and Properties of Confidence Sets for Ill-Posed Estimation Problems with Applications to Spectral Density and Persistence Estimation, Unit Roots, and Estimation of Long Memory Parameters." *Econometrica* 70(3), 1035–1065.
- Sun, Y. (2014). "Let's Fix It: Fixed-b Asymptotics versus Small-b Asymptotics in Heteroscedasticity and Autocorrelation Robust Inference." Journal of Econometrics 178(3), 659–677.
- Sun, Y., Phillips, P. C. B., and Jin, S. (2008). "Optimal Bandwidth Selection in Heteroskedasticity–Autocorrelation Robust Testing." *Econometrica* 76(1), 175–194.

Applications

Dew-Becker, I. (2017). "How Risky Is Consumption in the Long-Run? Benchmark Estimates from a Robust Estimator." *Review of Financial Studies* 30(2), 631–666.

6 Non-stationary models

Unit roots

* Hayashi: chapter 9.

Hamilton: chapters 15–17.

- Beveridge, S., and Nelson, C. R. (1981). "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'." *Journal of Monetary Economics* 7(2), 151–174.
- Elliott, G., Rothenberg, T. J., and Stock, J. H. (1996). "Efficient Tests for an Autoregressive Unit Root." *Econometrica* 64(4), 813–836.
- Hansen, B. E. (1999). "The Grid Bootstrap and the Autoregressive Model." *Review of Economics and Statistics* 81(4), 594–607.
- Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y. (1992). "Testing the null hypothesis of stationarity against the alternative of a unit root." *Journal of Econometrics* 54(1–3), 159–178.
- Mikusheva, A. (2007). "Uniform inference in autoregressive models." *Econometrica* 75(5), 1411–1452.
- Phillips, P. C. B. (1990). "To criticize the critics: An objective Bayesian analysis of stochastic trends." *Journal of Applied Econometrics* 6, 333–364. See also comments and rejoinder in the same journal issue.
- Phillips, P. C. B. (2012). "Folklore Theorems, Implicit Maps, and Indirect Inference." *Econometrica* 80(1), 425–454.
- Sims, C. A. (2000). "Using a likelihood perspective to sharpen econometric discourse: Three examples." *Journal of Econometrics* 95(2), 443–462. Section 2.

- Sims, C. A., and Uhlig, H. (1991). "Understanding Unit Rooters: A Helicopter Tour." *Econometrica* 59(6), 1591–1599.
- Stock, J. H. (1991). "Confidence intervals for the largest autoregressive root in U.S. macroeconomic time series." *Journal of Monetary Economics* 28(3), 435–459.
- Stock, J. H. (1994). "Unit roots, structural breaks and trends." In Handbook of Econometrics, Volume 4, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 46, 2739–2841. Sections 1–3, 6.

Detrending

- Baxter, M., and King, R. G. (1999). "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." *Review of Economics and Statistics* 81(4), 575–593.
- Christiano, L. J., and Fitzgerald, T. J. (2003). "The Band Pass Filter." *International Economic Review* 44(2), 435–465.
- Cogley, T., Nason, J. M. (1995). "Effects of the Hodrick-Prescott filter on trend and difference stationary time series: Implications for business cycle research." *Journal of Economic Dynamics and Control* 19(1), 253–278.
- Hamilton, J. D. (2017). "Why You Should Never Use the Hodrick-Prescott Filter." *Review* of *Economics and Statistics*, forthcoming.
- Hodrick, R. J., and Prescott, E. C. (1997). "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking* 29(1), 1–16.
- Ravn, M. O., and Uhlig, H. (2002). "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *Review of Economics and Statistics* 84(2), 371–376.

Cointegration

- * Hayashi: chapter 10.
- * Lütkepohl: chapters 6–8.
 - Hamilton: chapters 18–20.

- Elliott, G. (1998). "The Robustness of Cointegration Methods when Regressors Almost Have Unit Roots." *Econometrica* 66(1), 149–158.
- Watson, M. W. (1994). "Vector autoregressions and cointegration." In Handbook of Econometrics, Volume IV, edited by Engle, R. F., and McFadden, D. L., Elsevier, chapter 47, 2843–2915.

Long-run inference

- Müller, U. K., and Watson, M. W. (2008). "Testing Models of Low-Frequency Variability." *Econometrica* 76(5), 979–1016.
- Müller, U. K., and Watson, M. W. (2015), "Low-Frequency Econometrics." Manuscript, Princeton University.

Applications

- Johansen, S., and Juselius, K. (1992). "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and UIP of UK." *Journal of Econometrics* 53, 211– 244.
- King, R. G., Plosser, C. I., Stock, J. H., and Watson, M. W. (1991). "Stochastic Trends and Economic Fluctuations." *American Economic Review* 81(4), 819–840.
- Nelson, C. R., and Plosser, C. I. (1982). "Trends and random walks in macroeconomic time series." *Journal of Monetary Economics* 10(2), 139–162.
- Steinsson, J. (2008). "The Dynamic Behavior of the Real Exchange Rate in Sticky Price Models." American Economic Review 98(1), 519–533.